

UDC 330

RESIDUAL LIKELIHOOD APPROACH FOR ASYMMETRIC PRICE RELATIONSHIP SELECTION

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ABSTRACT

This study considers the problem of asymmetric price transmission model selection and investigates the performance of the recently developed model selection criteria (RIC) against commonly used alternatives (AIC and BIC) in terms of their ability to recover the true asymmetric data generating process. Asymmetric price transmission models are estimated and compared using the selection criteria. Monte Carlo simulation results indicate that the performance of the model selection methods depends on the sample size, the level of asymmetry and the amount of noise in the model used in the application. In larger samples, RIC is comparable to BIC and outperforms AIC. At higher noise levels, RIC is comparable to AIC and outperforms BIC. Additionally, at strong levels of asymmetry, RIC outperforms both AIC and BIC. These results suggest that RIC which has both BIC's useful property of consistency and efficient property of AIC is a very reliable and useful criterion in asymmetric price transmission model selection.

KEY WORDS

Price Asymmetry, Akaike's Information Criteria, Bayesian Information Criteria, Residual Information Criteria, residual likelihood, Monte Carlo Simulation, model selection.

One of the principal decisions faced in asymmetric price transmission modelling of agricultural and related markets is that of choosing an appropriate model from a list of competing models. To address this problem of choosing a single model out of all possible candidate models, various information criteria have been developed. Researchers involved in asymmetric price transmission modelling have employed the widely used Akaike Information Criteria (AIC) of Akaike (1973) and Bayesian Information Criteria (BIC) of Schwarz (1978) in addressing asymmetric price transmission model selection problems. For example, Ning and Sun (2014) investigated degree of vertical integration and the presence of asymmetric price transmission for sawtimber and lumber products in the southern and western United States using 4 models. In the end, they selected the MTAR model as the best model among the competing models on the basis of the AIC and BIC statistics. Acquah (2010) evaluates the relative performance of AIC and BIC in choosing between competing price transmission models and finds that AIC performed well in small samples whilst BIC, in contrast, performs relatively poorly in small samples but is consistent and performs well in large samples. In effect, AIC has the advantage of being efficient and BIC has the advantage of being consistent. The efficient property of AIC and consistent property of BIC is noted in previous studies (Markon and Krueger (2004) and Fishler et al. (2002)). A fundamental question which remains unanswered is how will a criteria that combines the AIC's efficient property and BIC's consistent property perform in addressing model selection issues in asymmetric price transmission modelling? Recently, a new criteria which combines the strength of AIC and BIC has been introduced by Shi and Tsai. Shi and Tsai (2002) proposed an interesting information criteria termed the Residual Information Criteria (RIC) based on the notion of the residual log likelihood function of the true and candidate model. RIC has the efficient property of AIC and the consistent property of BIC. In

effect, RIC combines the strengths of AIC and BIC. Previous studies have evaluated the performance of commonly used AIC and BIC in asymmetric price transmission modelling. There is the need to extend previous literature on comparison of model selection methods in asymmetric price transmission modelling to include the new criteria RIC which combines the strength of AIC and BIC. An important research question which remains unanswered in asymmetric price transmission model selection is how well will the recently developed RIC perform when compared with frequently used AIC and BIC in asymmetric price transmission model selection. Will RIC's performance be superior or comparable to AIC and BIC in asymmetric price transmission model selection? Will RIC point to the correct asymmetric price transmission model in price transmission analysis? Though Shi and Tsai's RIC has been successfully applied to a number of applications, such as normal linear regression, Box-Cox transformation, inverse regression models (Ni et al., 2005) and longitudinal data analysis (Li et al., 2005), its application in asymmetric price transmission analysis has not yet been explored. In effect little is known about its performance in asymmetric price transmission model selection. In order to address these issues, the problem of asymmetric price transmission model selection is considered and a Monte Carlo study is conducted to investigate the ability of the recently developed RIC and commonly used criteria (AIC and BIC) to clearly identify the correct asymmetry price transmission model out of the set of competing models. In effect, this study is aimed at evaluating the relative performance of the recently developed RIC against commonly used AIC and BIC in terms of their ability to recover the true asymmetric data generating process.

The model selection criteria can be used to rank the different asymmetric price transmission models used in modelling price asymmetry in agricultural markets. Based on the model ranking derived from the model selection criteria, a researcher can choose the most appropriate type of asymmetric behaviour (Houck asymmetry, Granger and Lee Asymmetry and Von Cramon and Loy asymmetry among others) that characterises the price transmission process in agricultural markets. Selection of the appropriate model leads to the correct asymmetric adjustment speeds or parameter estimates that describe the price transmission process and asymmetric behaviour in agricultural markets. The asymmetric adjustment parameters form the basis for detecting price asymmetry in agricultural markets.

The rest of the paper is presented as follows. In the following section, an introduction of the model selection criteria is presented. This is followed by a brief description of asymmetric price transmission models. A practical application in which the performance of the model selection methods in selecting the correct asymmetric model are evaluated via Monte Carlo experimentation is presented. Finally, the study ends with conclusions.

METHODS OF RESEARCH

Model Selection Using Information Criteria. Information criteria play an important role in model selection. Various information criteria have been proposed to be used in model selection. Most information criteria consist of two terms.

The first term is the negative log-likelihood, multiplied by two, of the data calculated with the maximum likelihood estimates of the parameters. The second term differs between different information criteria. The second term serves as a penalty for model complexity. In effect, it increases as the number of parameters in the model increases. In model selection using information criteria, the model that minimizes information criteria is declared as the best model among the set of models under consideration. In this section, commonly used information criteria such as AIC and BIC and recently developed Residual Information Criteria (RIC) are discussed.

Akaike Information Criterion (AIC). Akaike (1973) proposed the Akaike Information Criteria defined by:

$$AIC = -2 \log(L) + 2k \quad (1)$$

Where (L) denotes the maximum likelihood functions of the model with k covariates. The first term represents the log-likelihood of the model and the second term penalizes the model for complexity. AIC is asymptotically efficient (i.e. criteria select the best finite dimensional candidate model in large samples when the true model is of infinite dimension). AIC is motivated by the principle of minimizing Kullback-Leibler discrepancy. Thus AIC chooses the best-approximating model to the data generating process. Models with minimum AIC values are preferred.

Bayesian Information Criteria (BIC). Schwarz (1978) proposed the Bayesian Information Criteria (BIC) defined by:

$$BIC = -2 \log(L) + k \log(n) \quad (2)$$

Where (L) denotes the maximum likelihood function of the model with k covariates. The first term represents the log-likelihood of the model and the second term penalizes the model for complexity. BIC is asymptotically consistent (i.e. criteria select the correct model with probability approaching 1 in large samples when the true model is of finite dimension). BIC is motivated from Bayesian perspective and it selects the model with the maximum posterior probability for a given prior probability. Thus BIC chooses the true model. BIC differs from AIC in terms of the penalty term which is greater in BIC. Models with minimum BIC values are preferred.

Residual Information Criterion. Shi and Tsai (2002) recently introduced a new model selection criteria, Residual Information Criterion (RIC) that combines the strength of corrected AIC and BIC. RIC has both BIC's useful property of consistency and the efficient property of AIC. RIC is motivated by the principle of minimizing the Kullback-Leibler discrepancy between the residual log-likelihood functions of the true and candidate model. RIC is expressed as:

$$RIC = (n - k) \log(\sigma^2) + k \{ \log(n) - 1 \} + \frac{4}{n - k - 2} \quad (3)$$

Where the (σ^2) is the residual variance of the model, k is the number of parameters in the model and n is the sample size. Models with minimum RIC values are preferred.

Asymmetric Price Transmission Models. Houck (1977) developed an econometric method for investigating price asymmetry based on Wolfram (1971) price variable segmentation approach which can be represented as follows:

$$\Delta y_t = \beta_1^+ \Delta x_t^+ + \beta_1^- \Delta x_t^- + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2) \quad (4)$$

Where Δx_t^+ and Δx_t^- are the increases and decreases in the price series x_t . Asymmetry is tested by establishing whether the coefficients $(\beta_1^+$ and $\beta_1^-)$ are identical (that is $H_0 : \beta_1^+ = \beta_1^-$). The Houck's model assumes that the price series involved are not co-integrated.

Cramon-Taubadel and Loy (1996) and Cramon-Taubadel (1998) demonstrated that Houck's model is not an appropriate test for asymmetry if the price variables are co-integrated. In order to test for asymmetry in price series that are co-integrated, Granger and Lee (1989) proposed the Error Correction Model which can be represented as follows:

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2^+ ECT_{t-1}^+ + \beta_2^- ECT_{t-1}^- + \varepsilon_t \quad (5)$$

Where the price variables x and y are co-integrated and the long run equilibrium relationship between them is denoted by the Error Correction Term (ECT) which is decomposed by Wolfram-type segmentation into positive and negative component. In the Error Correction Model, asymmetries specified affect the positive and negative components of the error correction term. Symmetry in eq. (5) is tested by determining whether the coefficients (β_2^+ and β_2^-) are identical (that is $H_0 : \beta_2^+ = \beta_2^-$).

Cramon-Taubadel and Loy (1996) extend the Granger and Lee Model into a complex asymmetric price transmission model in which asymmetries specified affect the direct impact of price increases and decreases, as well as positive and negative components of the error correction term.

$$\Delta y_t = \beta_1^+ \Delta x_t^+ + \beta_1^- \Delta x_t^- + \beta_2^+ ECT_{t-1}^+ + \beta_2^- ECT_{t-1}^- + \varepsilon_t \quad (6)$$

Where Δx_t^+ and Δx_t^- are the increases and decreases in x_t and ECT_{t-1}^+ and ECT_{t-1}^- are the positive and negative components of the error correction term. The test for asymmetry using the above eq. 6 is: $H_0 : \beta_1^+ = \beta_1^-$ and $\beta_2^+ = \beta_2^-$. In effect, a joint F-test can be used to test symmetry or asymmetry of the price transmission process.

RESULTS AND DISCUSSION

Comparison of the Performance of Information Criteria. The empirical performance of a recently developed model selection criteria (RIC) and frequently used alternative model selection criteria (AIC and BIC) in recovering the true asymmetric data generating process (DGP) under conditions of different sample sizes and noise levels are evaluated using Monte Carlo studies. The data generating process is simulated from the standard error correction model in each Monte Carlo experiment as follows:

$$\Delta y_t = 0.7 \Delta x_t - 0.25(y_t - x_t)^+_{t-1} - 0.75(y_t - x_t)^-_{t-1} + \varepsilon \quad (7)$$

Drawing from the experimental designs of Holly et al (2003), the value of β_1 is set to 0.7 and $(\beta_2^+, \beta_2^-) \in (-0.25, -0.75)$ are considered for the coefficients of the standard asymmetric error correction terms in the true model. The prices y and x generated as I (1) nonstationary variables that are co-integrated and Δy_t in eq. 7 is obtained by summing up Δx_t and the positive and negative deviations from the equilibrium relationship between y and x .

The competing asymmetric price transmission models are fitted to the simulated data and the ability of the model selection methods to recover the true asymmetric data generating process was measured and defined as the model recovery rates. Model recovery rates were obtained using 1000 Monte Carlo simulations. The number of times each model selection criteria selects the true model provides the basis for comparison.

The relative performance of the recently developed Residual Information Criteria (RIC) and the commonly used alternative model selection criteria (AIC and BIC) are compared in terms of their ability to recover the true asymmetric data generating process (DGP) across various sample size conditions and noise levels (i.e. Model Recovery Rates). In order to ensure brevity, the standard asymmetric error correction model, the complex asymmetric error correction model and the Houck's model in first differences are denoted by SECM, CECM, and HKD respectively.

The performance of the model selection methods is evaluated in terms of their ability to select the true model among a set of competing models. Table 1 reports the success rates with which each model selection criteria selects the true model. The model selection methods studied were capable of identifying the true model, though their ability to recover the true asymmetric data generating process (DGP) increases with increase in sample size and a decrease in stochastic variance. Similarly, previous studies (Acquah 2013; Markon & Krueger, 2004; Bozdogan, 1987; Atkinson, 1980) also noted that model selection methods empirically do point to the correct model. Generally, as sample size increases, model recovery rates of RIC, BIC, and AIC improved. In small samples (upper part of Table 1), the model selection methods recovered at most 81.8 % of the true data generating process.

In moderate samples (middle part of Table 1), the model selection methods recovered at most 97.1 % of the true data generating process. When the sample size was large (lower part of Table 1), the model selection methods recovered at most 99.1 % of the true model. Generally RIC performs similarly to BIC in small to large samples with low noise levels and both outperform AIC in small to large samples with low noise levels. In effect, at larger sample size, RIC is comparable to BIC and outperforms AIC.

Recovery rates of Residual Information Criteria strongly depended on sample size for the true data generating process (DGP). It increased from 81.6 percent to 99.0 percent when the sample size was increased from 50 to 500. Similarly, recovery rates of Bayesian Information Criteria also varied strongly with sample size for the true data generating process (DGP). It increased from 81.8 percent to 99.1 percent when the sample size was increased from 50 to 500. This is consistent with the empirical studies which indicate that BIC or RIC is consistent (Markon and Krueger (2004); Fishler et al. (2002); Shi and Tsai (2002).

On the other hand, recovery rates of AIC increased from 78.7 percent to 85.0 percent for the true asymmetric data generating process (DGP) when the sample size was increased from 50 to 500. Though AIC performed well in the small samples, it is inconsistent and does not improve in performance as sample size increases. Similarly, Markon and Krueger (2004) note that AIC performs well in small samples but does not improve in performance in large samples and BIC performs poorly in small samples but improves performance in large samples. Notably, the poor performance of AIC in large samples is better explained by the fact that it exhibits a tendency to select the complex model (15%) as demonstrated in the lower part of Table 1. Acquah (2010) also found that the ability of AIC to select a true model rapidly increased with sample size but at larger sample sizes it continued to exhibit a slight tendency to select complex models whilst BIC, in contrast, is consistent and improves in performance as sample size increased. Shi and Tsai (2002) noted that RIC performs similarly to BIC and outperforms AIC when the sample size is large. Generally, these results are confirmed in the Monte Carlo simulation results presented in Table 1.

The effects of noise level on model selection are studied. Three error sizes (σ) ranging from small to large and corresponding to 1.0, 2.0 and 3.0 are considered in the study. Using 1000 simulations via Monte Carlo experimentation, data is generated from equation (7) with the different error sizes and a sample size of 150. The competing models are compared to the true model on the basis of their data fitting abilities as the error in the data generating process was increased systematically. Table 2 shows the percentage with which each model selection

criteria selects the true asymmetric data generating process (i.e. SECM) among a set of competing models as the amount of noise in the data generating process was increased.

Table 1 – Relative performance of the model selection methods across sample size

Experiment criterion	Model fitted			
	Methods	CECM (%)	HKD (%)	SECM (DGP) (%)
$n = 50$ $\sigma = 1$	AIC	16.2	5.1	78.7
	BIC	5.2	13	81.8
	RIC	4.2	14.2	81.6
$n = 150$ $\sigma = 1$	AIC	15.6	0	84.4
	BIC	3.2	0	96.8
	RIC	2.9	0	97.1
$n = 500$ $\sigma = 1$	AIC	15	0	85
	BIC	0.9	0	99.1
	RIC	1	0	99

Note: Recovery rates based on 1000 replications.

Table 2 – Relative performance of the selection methods across error size

Experiment criterion	Model fitted			
	Methods	CECM (%)	HKD (%)	SECM (DGP) (%)
$n = 150$ $\sigma = 3$	AIC	12.1	22.8	65.1
	BIC	1.6	55.6	42.8
	RIC	6.2	33	60.8
$n = 150$ $\sigma = 2$	AIC	14.5	4.8	80.7
	BIC	2.1	18.2	79.7
	RIC	4.4	11.1	84.5
$n = 150$ $\sigma = 1$	AIC	15.6	0	84.4
	BIC	3.2	0	96.8
	RIC	2.9	0	97.1

Note: Recovery rates percentages based on 1000 replications.

Generally, as the amount of noise in the true asymmetric price transmission data generating process increased, model selection performance declined. Recovery rates of Residual Information Criteria decreased from 97.1 percent to 60.8 percent when the error size was increased from 1 to 3. Similarly, recovery rates of BIC decreased from 96.8 percent to 42.8 percent for the true data generating process (DGP) when the error size was increased from 1 to 3. Recovery rates of AIC also decreased from 84.4 percent to 65.1 percent for the true asymmetric data generating process (DGP) when the error size was increased from 1 to 3. These results are generally consistent with previous studies (See Acquah, 2013; Myung, 2000; Gheissari and Bab-Hadiashar, 2004; Yang, 2003) which found that the recovery rates of the true data generating process decreases with increasing noise levels. In effect, at higher noise levels, RIC is comparable to AIC and outperforms BIC.

Simulating the effects of sample size and stochastic variance concurrently indicates that a small error and large sample improves recovery of the true asymmetric data generating process and vice versa as illustrated in Table 3. With a small sample of 50 and an error size of 2.0, the true data generating process was recovered at least 41.2 percent of the time by the model selection criteria as illustrated in the upper part of Table 3. On the other hand, with a relatively large sample of 150 and error size of 0.5, at least 84.4 percent of the correct model was recovered across all the model selection methods as indicated in the lower part of Table 3.

The model recovery rates of the model selection methods are derived under combined conditions of a small sample size of 50 and large error size of 2 (i.e. Unstable conditions), and a relatively large sample size of 150 and a small error size of 0.5 (i.e. Stable conditions).

In effect under stable conditions, model selection performance or recovery rates improved whilst under unstable conditions, model selection performance or recovery rates decreased.

Table 3 – Effects of sample size and stochastic variance on model recovery

Experiment criterion	Methods	Model fitted		
		CECM (%)	HKD (%)	SECM (DGP) (%)
$\sigma = 2 \quad n = 50$	AIC	10.8	37.5	51.7
	BIC	2.3	56.5	41.2
	RIC	6	43	51
$n = 150 \quad \sigma = 0.5$	AIC	15.6	0	84.4
	BIC	3.2	0	96.8
	RIC	0.8	0	99.2

Note: Recovery rates based on 1000 replications.

At higher noise levels and small sample size (unstable conditions), RIC is similar to AIC and outperforms BIC. At lower noise levels and large sample size (stable conditions), RIC is similar to BIC and outperforms AIC. Similarly, Chen et al. (2007) notes the tendency of BIC to perform worse than AIC at high noise levels in a factorial analysis. In a comparison of model selection methods, Yang (2003) demonstrates that AIC outperforms BIC in recovering the true model as noise levels increased in a linear regression analysis framework.

The results are generally consistent with trend suggested by previous Monte Carlo experimentation (Acquah, 2010) which suggest that the recovery rates of the true data generating process decreased with increasing noise levels in asymmetric price transmission regression models.

The effects of level of asymmetry on model selection are investigated by simulating data of sample size 150 and error size 1 from the standard error correction model and considering asymmetry values of $(\beta_2^+, \beta_2^-) \in (-0.25, -0.50)$ or $(-0.25, -0.75)$ for the coefficients of the asymmetric error correction terms. The effect of the increase in difference of asymmetric adjustment parameters on model recovery is then investigated. Increasing the difference in the asymmetric adjustment parameters from 0.25 to 0.50 results in an increase in model recovery of the true asymmetric data generating process as illustrated in Table 4.

Table 4 – Effects of the level of asymmetry on model recovery

Experiment criterion	Methods	Model Fitted		
		CECM (%)	HKD (%)	SECM (DGP) (%)
$\beta_2^+ - \beta_2^- = 0.25$	AIC	15.7	0	84.3
	BIC	2.9	1.4	95.7
	RIC	2.6	1.5	95.9
$\beta_2^+ - \beta_2^- = 0.50$	AIC	15.6	0	84.4
	BIC	3.2	0	96.8
	RIC	2.9	0	97.1

Note: Recovery rates percentages based on 1000 replications

Recovery rates of the Residual Information Criteria respond more strongly to increases in the difference between the asymmetric adjustment parameters than other criteria (AIC and BIC). Cook et al. (1999) without regards to the concept of information criteria note that the increases in the difference in asymmetric adjustment parameters from 0.25 to 0.50 have positive effects on the test for asymmetry. Notably, the performance of the model selection methods in

recovering the true data generating process depends on the difference in asymmetric adjustment parameters or speeds.

CONCLUSION

This study considered the problem of asymmetric price transmission model selection and investigated the ability of the recently developed Residual Information Criteria (RIC) and commonly used criteria (AIC and BIC) to clearly identify the correct asymmetric price transmission model out of a set of competing models via Monte Carlo experimentation. The Monte Carlo simulation results indicated that the sample sizes, noise levels and the level of asymmetry are essential in the selection of the true asymmetric price transmission model. Large sample sizes or low noise levels improve the ability of the model selection methods to identify the correct asymmetric price data generating process. Generally, RIC's performance is similar to BIC and outperforms AIC under stable conditions such as a large sample and small noise levels. Under unstable conditions such as a small sample and high noise levels, RIC's performance is similar to AIC and outperforms BIC. At larger sample size, RIC is comparable to BIC and outperforms AIC. At higher noise levels, RIC is comparable to AIC and outperforms BIC. At strong levels of asymmetry, RIC outperforms both AIC and BIC. These results suggest that RIC, which combines the strengths of AIC and BIC is a very reliable and useful criterion in asymmetric price transmission model selection.

The empirical comparison provided contributes to knowledge and understanding of the relative performance of recently developed RIC against commonly used AIC and BIC in an asymmetric price transmission modelling framework. The study also adds to the literature on asymmetric price transmission modelling by drawing the attention and interests of asymmetric price transmission researchers to adopt more recent statistical model selection criteria, such as RIC, in asymmetric price transmission model selection problems.

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